THINKING KIDS

Singapore MATH

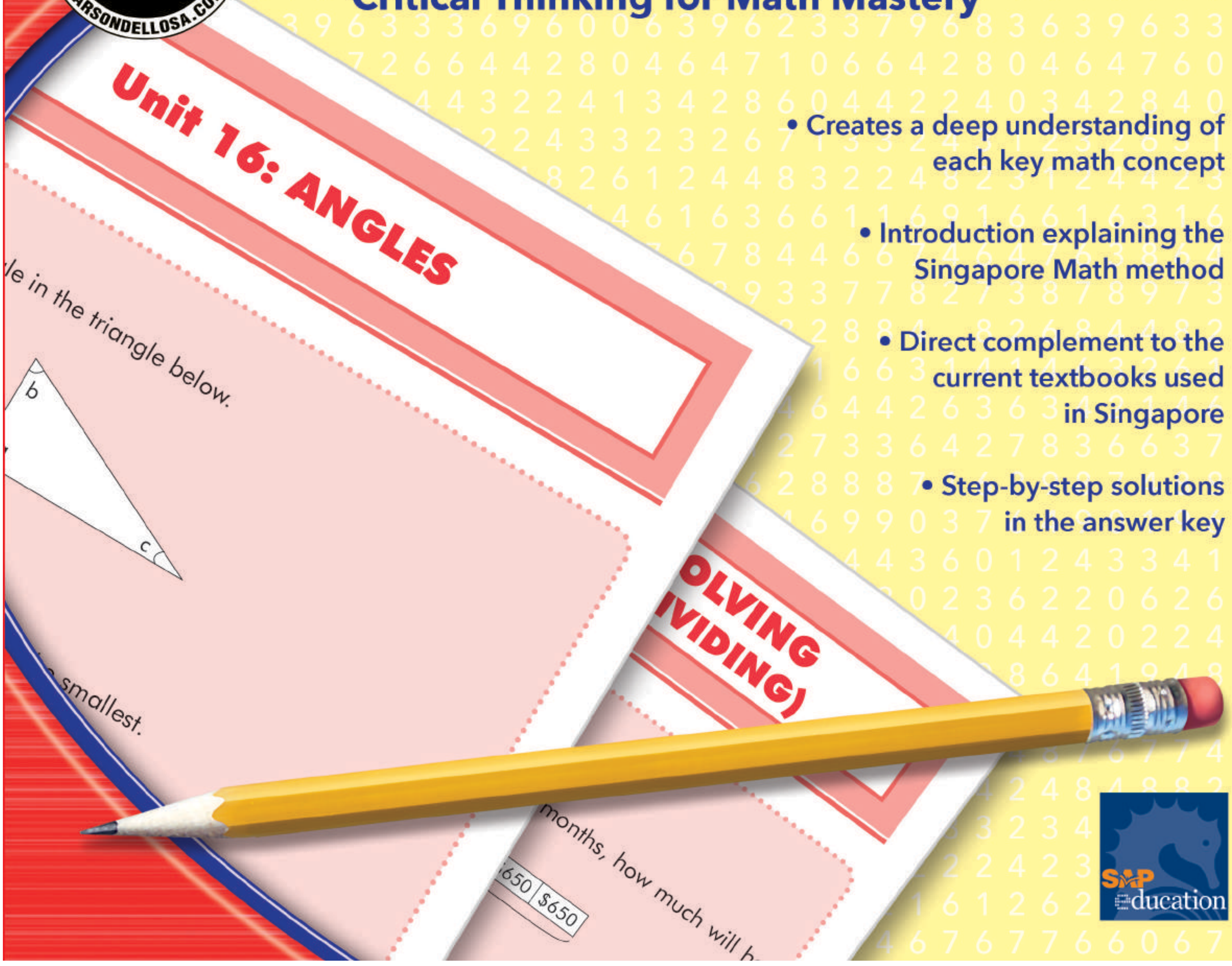
LEVEL 3 A&B

Appropriate for students in Grade 4

CORRELATED TO Current State Standards CARSONDELLOSA.COM

Boost Problem Solving and Critical Thinking for Math Mastery

- Creates a deep understanding of each key math concept
- Introduction explaining the Singapore Math method
- Direct complement to the current textbooks used in Singapore
- Step-by-step solutions in the answer key



SMP education

INTRODUCTION TO SINGAPORE MATH

Welcome to Singapore Math! The math curriculum in Singapore has been recognized worldwide for its excellence in producing students highly skilled in mathematics. Students in Singapore have ranked at the top in the world in mathematics on the *Trends in International Mathematics and Science Study* (TIMSS) in 1993, 1995, 2003, and 2008. Because of this, Singapore Math has gained in interest and popularity in the United States.

Singapore Math curriculum aims to help students develop the necessary math concepts and process skills for everyday life and to provide students with the ability to formulate, apply, and solve problems. Mathematics in the Singapore Primary (Elementary) Curriculum cover fewer topics but in greater depth. Key math concepts are introduced and built on to reinforce various mathematical ideas and thinking. Students in Singapore are typically one grade level ahead of students in the United States.

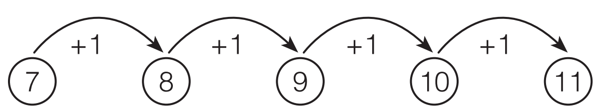
The following pages provide examples of the various math problem types and skill sets taught in Singapore.

At an elementary level, some simple mathematical skills can help students understand mathematical principles. These skills are the counting-on, counting-back, and crossing-out methods. Note that these methods are most useful when the numbers are small.

1. The Counting-On Method

Used for addition of two numbers. Count on in 1s with the help of a picture or number line.

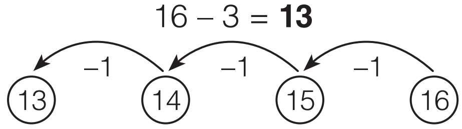
7 + 4 = **11**



2. The Counting-Back Method

Used for subtraction of two numbers. Count back in 1s with the help of a picture or number line.

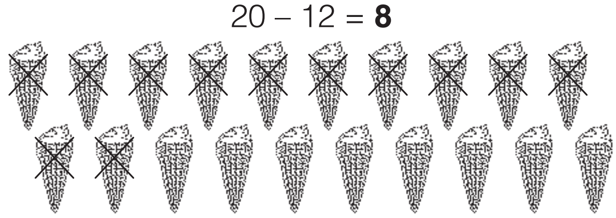
16 – 3 = **13**



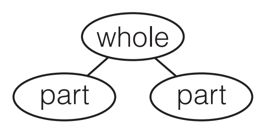
3. The Crossing-Out Method

Used for subtraction of two numbers. Cross out the number of items to be taken away. Count the remaining ones to find the answer.

20 – 12 = **8**



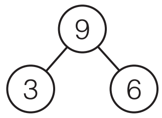
A **number bond** shows the relationship in a simple addition or subtraction problem. The number bond is based on the concept “part-part-whole.” This concept is useful in teaching simple addition and subtraction to young children.



To find a whole, students must add the two parts.
To find a part, students must subtract the other part from the whole.

The different types of number bonds are illustrated on the next page.

1. Number Bond (single digits)

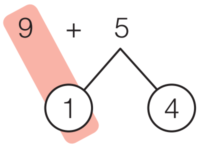


3 (part) + 6 (part) = **9** (whole)

9 (whole) – 3 (part) = **6** (part)

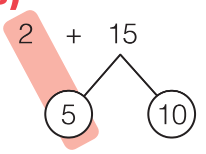
9 (whole) – 6 (part) = **3** (part)

2. Addition Number Bond (single digits)



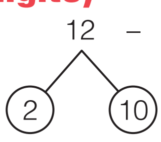
= 9 + 1 + 4 Make a ten first.
= 10 + 4
= **14**

3. Addition Number Bond (double and single digits)



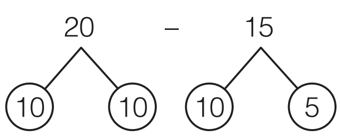
= 2 + 5 + 10 Regroup 15 into 5 and 10.
= 7 + 10
= **17**

4. Subtraction Number Bond (double and single digits)



10 – 7 = 3
3 + 2 = **5**

5. Subtraction Number Bond (double digits)



10 – 5 = 5
10 – 10 = 0
5 + 0 = **5**

Students should understand that multiplication is repeated addition and that division is the grouping of all items into equal sets.

1. Repeated Addition (Multiplication)

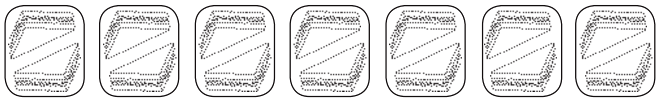
Mackenzie eats 2 rolls a day. How many rolls does she eat in 5 days?

$$2 + 2 + 2 + 2 + 2 = 10$$
$$5 \times 2 = 10$$

She eats **10** rolls in 5 days.

2. The Grouping Method (Division)

Mrs. Lee makes 14 sandwiches. She gives all the sandwiches equally to 7 friends. How many sandwiches does each friend receive?



$$14 \div 7 = 2$$

Each friend receives **2** sandwiches.

One of the basic but essential math skills students should acquire is to perform the 4 operations of whole numbers and fractions. Each of these methods is illustrated below.

1. The Adding-Without-Regrouping Method

	H	T	O
	3	2	1
+	5	6	8
	8	**8**	**9**

O: Ones
T: Tens
H: Hundreds

Since no regrouping is required, add the digits in each place value accordingly.

2. The Adding-by-Regrouping Method

	H	T	O
	[1]4	9	2
+	1	5	3
	6	**4**	**5**

O: Ones
T: Tens
H: Hundreds

In this example, regroup 14 tens into 1 hundred 4 tens.

3. The Adding-by-Regrouping-Twice Method

$$\begin{array}{rccc} & H & T & O \\ & {}^{1}2 & {}^{1}8 & 6 \\ + & 3 & 6 & 5 \\ \hline & \mathbf{6} & \mathbf{5} & \mathbf{1} \end{array}$$

O: Ones
T: Tens
H: Hundreds

Regroup twice in this example.
First, regroup 11 ones into 1 ten 1 one.
Second, regroup 15 tens into 1 hundred 5 tens.

4. The Subtracting-Without-Regrouping Method

$$\begin{array}{rccc} & H & T & O \\ & 7 & 3 & 9 \\ - & 3 & 2 & 5 \\ \hline & \mathbf{4} & \mathbf{1} & \mathbf{4} \end{array}$$

O: Ones
T: Tens
H: Hundreds

Since no regrouping is required, subtract the digits in each place value accordingly.

5. The Subtracting-by-Regrouping Method

$$\begin{array}{rccc} & H & T & O \\ & 5 & {}^{7}\cancel{8} & {}^{11}\cancel{1} \\ - & 2 & 4 & 7 \\ \hline & \mathbf{3} & \mathbf{3} & \mathbf{4} \end{array}$$

O: Ones
T: Tens
H: Hundreds

In this example, students cannot subtract 7 ones from 1 one. So, regroup the tens and ones. Regroup 8 tens 1 one into 7 tens 11 ones.

6. The Subtracting-by-Regrouping-Twice Method

$$\begin{array}{rccc} & H & T & O \\ & {}^{7}\cancel{8} & {}^{9}\cancel{0} & {}^{10}\cancel{0} \\ - & 5 & 9 & 3 \\ \hline & \mathbf{2} & \mathbf{0} & \mathbf{7} \end{array}$$

O: Ones
T: Tens
H: Hundreds

In this example, students cannot subtract 3 ones from 0 ones and 9 tens from 0 tens. So, regroup the hundreds, tens, and ones. Regroup 8 hundreds into 7 hundreds 9 tens 10 ones.

7. The Multiplying-Without-Regrouping Method

$$\begin{array}{rcc} & T & O \\ & 2 & 4 \\ \times & & 2 \\ \hline & \mathbf{4} & \mathbf{8} \end{array}$$

O: Ones
T: Tens

Since no regrouping is required, multiply the digit in each place value by the multiplier accordingly.

8. The Multiplying-With-Regrouping Method

$$\begin{array}{rccc} & H & T & O \\ & {}^{1}3 & {}^{2}4 & 9 \\ \times & & & 3 \\ \hline \mathbf{1,} & \mathbf{0} & \mathbf{4} & \mathbf{7} \end{array}$$

O: Ones
T: Tens
H: Hundreds

In this example, regroup 27 ones into 2 tens 7 ones, and 14 tens into 1 hundred 4 tens.

9. The Dividing-Without-Regrouping Method

$$\begin{array}{r} \mathbf{2\ 4\ 1} \\ 2\overline{)\,4\ 8\ 2} \\ -4 \\ \hline 8 \\ -8 \\ \hline 2 \\ -2 \\ \hline 0 \end{array}$$

Since no regrouping is required, divide the digit in each place value by the divisor accordingly.

10. The Dividing-With-Regrouping Method

$$\begin{array}{r} \mathbf{1\ 6\ 6} \\ 5\overline{)\,8\ 3\ 0} \\ -5 \\ \hline 3\ 3 \\ -3\ 0 \\ \hline 3\ 0 \\ -3\ 0 \\ \hline 0 \end{array}$$

In this example, regroup 3 hundreds into 30 tens and add 3 tens to make 33 tens. Regroup 3 tens into 30 ones.

11. The Addition-of-Fractions Method

$$\frac{1\times2}{6\times2}+\frac{1\times3}{4\times3}=\frac{2}{12}+\frac{3}{12}=\mathbf{\frac{5}{12}}$$

Always remember to make the denominators common before adding the fractions.

12. The Subtraction-of-Fractions Method

$$\frac{1\times5}{2\times5}-\frac{1\times2}{5\times2}=\frac{5}{10}-\frac{2}{10}=\mathbf{\frac{3}{10}}$$

Always remember to make the denominators common before subtracting the fractions.

13. The Multiplication-of-Fractions Method

$$\frac{{}^{1}\cancel{3}}{5}\times\frac{1}{{}_{3}\cancel{9}}=\mathbf{\frac{1}{15}}$$

When the numerator and the denominator have a common multiple, reduce them to their lowest fractions.

14. The Division-of-Fractions Method

$$\frac{7}{9}\div\frac{1}{6}=\frac{7}{{}_{3}\cancel{9}}\times\frac{\cancel{6}^{2}}{1}=\frac{14}{3}=\mathbf{4\frac{2}{3}}$$

When dividing fractions, first change the division sign (÷) to the multiplication sign (×). Then, switch the numerator and denominator of the fraction on the right hand side. Multiply the fractions in the usual way.

Model drawing is an effective strategy used to solve math word problems. It is a visual representation of the information in word problems using bar units. By drawing the models, students will know of the variables given in the problem, the variables to find, and even the methods used to solve the problem.

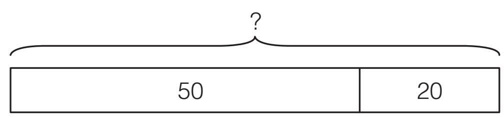
Drawing models is also a versatile strategy. It can be applied to simple word problems involving addition, subtraction, multiplication, and division. It can also be applied to word problems related to fractions, decimals, percentage, and ratio.

The use of models also trains students to think in an algebraic manner, which uses symbols for representation.

The different types of bar models used to solve word problems are illustrated below.

1. The model that involves addition

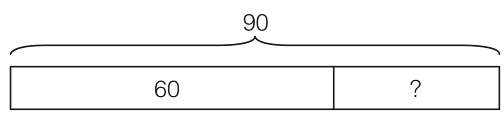
Melissa has 50 blue beads and 20 red beads. How many beads does she have altogether?



50 + 20 = **70**

2. The model that involves subtraction

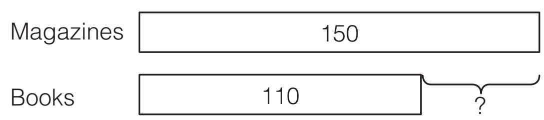
Ben and Andy have 90 toy cars. Andy has 60 toy cars. How many toy cars does Ben have?



90 – 60 = **30**

3. The model that involves comparison

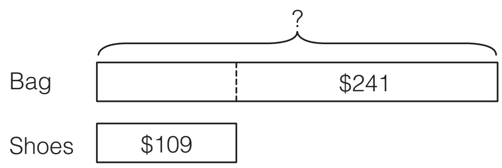
Mr. Simons has 150 magazines and 110 books in his study. How many more magazines than books does he have?



150 – 110 = **40**

4. The model that involves two items with a difference

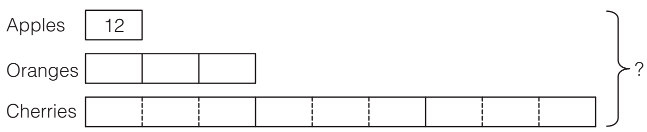
A pair of shoes costs \$109. A leather bag costs \$241 more than the pair of shoes. How much is the leather bag?



\$109 + \$241 = **\$350**

5. The model that involves multiples

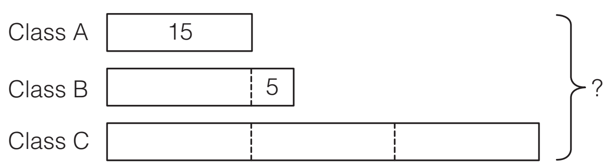
Mrs. Drew buys 12 apples. She buys 3 times as many oranges as apples. She also buys 3 times as many cherries as oranges. How many pieces of fruit does she buy altogether?



13×12 = **156**

6. The model that involves multiples and difference

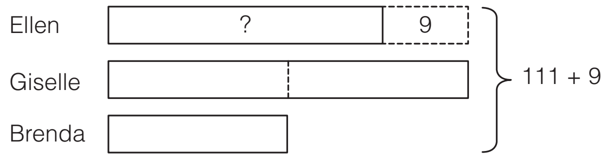
There are 15 students in Class A. There are 5 more students in Class B than in Class A. There are 3 times as many students in Class C than in Class A. How many students are there altogether in the three classes?



$(5 \times 15) + 5$ = **80**

7. The model that involves creating a whole

Ellen, Giselle, and Brenda bake 111 muffins. Giselle bakes twice as many muffins as Brenda. Ellen bakes 9 fewer muffins than Giselle. How many muffins does Ellen bake?

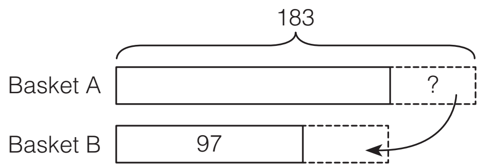


$(111 + 9) \div 5 = 24$

$(2 \times 24) - 9$ = **39**

8. The model that involves sharing

There are 183 tennis balls in Basket A and 97 tennis balls in Basket B. How many tennis balls must be transferred from Basket A to Basket B so that both baskets contain the same number of tennis balls?

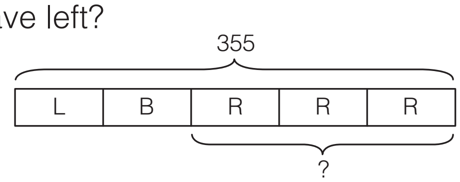


$183 - 97 = 86$

$86 \div 2$ = **43**

9. The model that involves fractions

George had 355 marbles. He lost $\frac{1}{5}$ of the marbles and gave $\frac{1}{4}$ of the remaining marbles to his brother. How many marbles did he have left?



L: Lost
B: Brother
R: Remaining

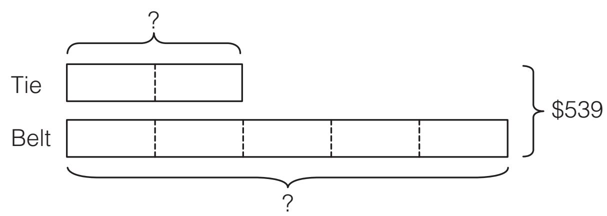
5 parts → 355 marbles
1 part → $355 \div 5 = 71$ marbles
3 parts → 3×71 = **213** marbles

10. The model that involves ratio

Aaron buys a tie and a belt. The prices of the tie and belt are in the ratio 2 : 5. If both items cost \$539,

(a) what is the price of the tie?

(b) what is the price of the belt?



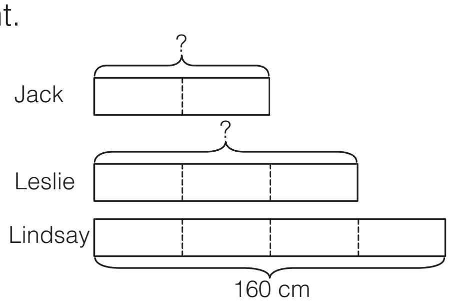
$539 ÷ 7 = $77

Tie (2 units) → 2 x \$77 = **\$154**

Belt (5 units) → 5 x \$77 = **\$385**

11. The model that involves comparison of fractions

Jack's height is $\frac{2}{3}$ of Leslie's height. Leslie's height is $\frac{3}{4}$ of Lindsay's height. If Lindsay is 160 cm tall, find Jack's height and Leslie's height.



1 unit → 160 ÷ 4 = 40 cm

Leslie's height (3 units) → 3 × 40 = **120 cm**

Jack's height (2 units) → 2 × 40 = **80 cm**

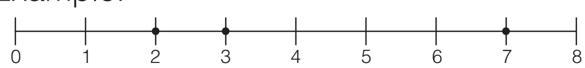
Thinking skills and strategies are important in mathematical problem solving. These skills are applied when students think through the math problems to solve them. The following are some commonly used thinking skills and strategies applied in mathematical problem solving.

1. Comparing

Comparing is a form of thinking skill that students can apply to identify similarities and differences.

When comparing numbers, look carefully at each digit before deciding if a number is greater or less than the other. Students might also use a number line for comparison when there are more numbers.

Example:



3 is greater than 2 but smaller than 7.

2. Sequencing

A sequence shows the order of a series of numbers. *Sequencing* is a form of thinking skill that requires students to place numbers in a particular order. There are many terms in a sequence. The terms refer to the numbers in a sequence.

To place numbers in a correct order, students must first find a rule that generates the sequence. In a simple math sequence, students can either add or subtract to find the unknown terms in the sequence.

Example: Find the 7th term in the sequence below.

1,	4,	7,	10,	13,	16	?
1st term	2nd term	3rd term	4th term	5th term	6th term	7th term

Step 1: This sequence is in an increasing order.

Step 2: 4 – 1 = 3 7 – 4 = 3
The difference between two consecutive terms is 3.

Step 3: 16 + 3 = 19
The 7th term is **19**.

3. Visualization

Visualization is a problem solving strategy that can help students visualize a problem through the use of physical objects. Students will play a more active role in solving the problem by manipulating these objects.

The main advantage of using this strategy is the mobility of information in the process of solving the problem. When students make a wrong step in the process, they can retrace the step without erasing or canceling it.

The other advantage is that this strategy helps develop a better understanding of the problem or solution through visual objects or images. In this way, students will be better able to remember how to solve these types of problems.

Some of the commonly used objects for this strategy are toothpicks, straws, cards, strings, water, sand, pencils, paper, and dice.

4. Look for a Pattern

This strategy requires the use of observational and analytical skills. Students have to observe the given data to find a pattern in order to solve the problem. Math word problems that involve the use of this strategy usually have repeated numbers or patterns.

Example: Find the sum of all the numbers from 1 to 100.

Step 1: <u>Simplify the problem.</u>

Find the sum of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

Step 2: <u>Look for a pattern.</u>

$1 + 10 = 11 \qquad 2 + 9 = 11$

$3 + 8 = 11 \qquad 4 + 7 = 11$

$5 + 6 = 11$

Step 3: <u>Describe the pattern.</u>

When finding the sum of 1 to 10, add the first and last numbers to get a result of 11. Then, add the second and second last numbers to get the same result. The pattern continues until all the numbers from 1 to 10 are added. There will be 5 pairs of such results. Since each addition equals 11, the answer is then $5 \times 11 = 55$.

Step 4: <u>Use the pattern to find the answer.</u>

Since there are 5 pairs in the sum of 1 to 10, there should be ($10 \times 5 = 50$ pairs) in the sum of 1 to 100.

Note that the addition for each pair is not equal to 11 now. The addition for each pair is now ($1 + 100 = 101$).

$50 \times 101 = 5050$

The sum of all the numbers from 1 to 100 is **5,050**.

5. Working Backward

The strategy of working backward applies only to a specific type of math word problem. These word problems state the end result, and students are required to find the total number. In order to solve these word problems, students have to work backward by thinking through the correct sequence of events. The strategy of working backward allows students to use their logical reasoning and sequencing to find the answers.

Example: Sarah has a piece of ribbon. She cuts the ribbon into 4 equal parts. Each part is then cut into 3 smaller equal parts. If the length of each small part is 35 cm, how long is the piece of ribbon?

$3 \times 35 = 105$ cm

$4 \times 105 = 420$ cm

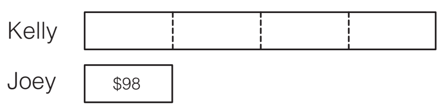
The piece of ribbon is **420 cm**.

6. The Before-After Concept

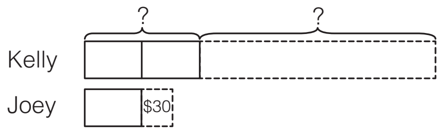
The *Before-After* concept lists all the relevant data before and after an event. Students can then compare the differences and eventually solve the problems. Usually, the Before-After concept and the mathematical model go hand in hand to solve math word problems. Note that the Before-After concept can be applied only to a certain type of math word problem, which trains students to think sequentially.

Example: Kelly has 4 times as much money as Joey. After Kelly uses some money to buy a tennis racquet, and Joey uses $30 to buy a pair of pants, Kelly has twice as much money as Joey. If Joey has $98 in the beginning,
(a) how much money does Kelly have in the end?
(b) how much money does Kelly spend on the tennis racquet?

Before



After



(a) $98 - $30 = $68
2 × $68 = $136
Kelly has **$136** in the end.

(b) 4 × $98 = $392
$392 – $136 = $256
Kelly spends **$256** on the tennis racquet.

7. Making Supposition

Making supposition is commonly known as "making an assumption." Students can use this strategy to solve certain types of math word problems. Making assumptions will eliminate some possibilities and simplifies the word problems by providing a boundary of values to work within.

Example: Mrs. Jackson bought 100 pieces of candy for all the students in her class. How many pieces of candy would each student receive if there were 25 students in her class?

In the above word problem, assume that each student received the same number of pieces. This eliminates the possibilities that some students would receive more than others due to good behavior, better results, or any other reason.

8. Representation of Problem

In problem solving, students often use representations in the solutions to show their understanding of the problems. Using representations also allow students to understand the mathematical concepts and relationships as well as to manipulate the information presented in the problems. Examples of representations are diagrams and lists or tables.

Diagrams allow students to consolidate or organize the information given in the problems. By drawing a diagram, students can see the problem clearly and solve it effectively.

A list or table can help students organize information that is useful for analysis. After analyzing, students can then see a pattern, which can be used to solve the problem.

9. Guess and Check

One of the most important and effective problem-solving techniques is *Guess and Check*. It is also known as *Trial and Error*. As the name suggests, students have to guess the answer to a problem and check if that guess is correct. If the guess is wrong, students will make another guess. This will continue until the guess is correct.

It is beneficial to keep a record of all the guesses and checks in a table. In addition, a *Comments* column can be included. This will enable students to analyze their guess (if it is too high or too low) and improve on the next guess. Be careful; this problem-solving technique can be tiresome without systematic or logical guesses.

Example: Jessica had 15 coins. Some of them were 10-cent coins and the rest were 5-cent coins. The total amount added up to $1.25. How many coins of each kind were there?

Use the guess-and-check method.

Number of 10¢ Coins	Value	Number of 5¢ Coins	Value	Total Number of Coins	Total Value
7	7 × 10¢ = 70¢	8	8 × 5¢ = 40¢	7 + 8 = 15	70¢ + 40¢ = 110¢ = $1.10
8	8 × 10¢ = 80¢	7	7 × 5¢ = 35¢	8 + 7 = 15	80¢ + 35¢ = 115¢ = $1.15
10	10 × 10¢ = 100¢	5	5 × 5¢ = 25¢	10 + 5 = 15	100¢ + 25¢ = 125¢ = $1.25

There were **ten** 10-cent coins and **five** 5-cent coins.

10. Restate the Problem

When solving challenging math problems, conventional methods may not be workable. Instead, restating the problem will enable students to see some challenging problems in a different light so that they can better understand them.

The strategy of restating the problem is to "say" the problem in a different and clearer way. However, students have to ensure that the main idea of the problem is not altered.

How do students restate a math problem?

First, read and understand the problem. Gather the given facts and unknowns. Note any condition(s) that have to be satisfied.

Next, restate the problem. Imagine narrating this problem to a friend. Present the given facts, unknown(s), and condition(s). Students may want to write the "revised" problem. Once the "revised" problem is analyzed, students should be able to think of an appropriate strategy to solve it.

11. Simplify the Problem

One of the commonly used strategies in mathematical problem solving is simplification of the problem. When a problem is simplified, it can be "broken down" into two or more smaller parts. Students can then solve the parts systematically to get to the final answer.