Read and write proper fractions, mixed numbers,

and improper fractions in standard and

Lesson Introduction: The Meaning of Fractions

Common Core State Standard

- Fifth Grade: 5.NF.3-7
- Sixth Grade: 6.NS.1, 6.NS.4
- Seventh Grade: 7.NS.1
- Eighth Grade: 8.NS.1

Vocabulary

denominator, fraction, fraction bar, improper fractions, mixed number, numerator, proper fraction, standard form, whole number, word form

Objective

word form.

Overview

There are three types of fractions: proper fractions, improper fractions, and mixed numbers.

A **proper fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. The bottom number of a fraction is the **denominator**. It tells you how many equal parts the whole is divided into. The top number of the fraction is the **numerator**. It tells how many equal parts of the whole you have. The numerator is less than the denominator in a proper fraction. A **fraction bar** is a line separating the numerator and denominator of a fraction and stands for division.

An **improper fraction** is a fraction in which the numerator is greater than or equal to the denominator.

A **mixed number** is a whole number and a fraction. A **whole number** is a counting number.

A fraction can be written in **standard form** or **word form**. A fraction written in standard form is written as a number. A fraction written in word form is written as words.

Three types of Fractions





Real-World Connection

A pizza cook has to be able to read and understand fractions. Example: A customer's order might read "large pizza, 1/2 pepperoni, 2/5 mushrooms and 1/10 bacon."

Lesson Introduction: Simplifying Fractions

Common Core State Standard

- Fifth Grade: 5.NF.3-7
- Sixth Grade: 6.NS.1, 6.NS.4
- Seventh Grade: 7.NS.1
- Eighth Grade: 8.NS.1

Objective Write fractions in lowest terms.

- Find the factors of numbers.
- Find the greatest common factor of two numbers.

Vocabulary

common factor, equivalent, factors, greatest common factor, simplest form, simplify

Overview

Every number has factors. The **factors** of a number divide that number evenly. The factors of 18 are 1, 2, 3, 6, 9, and 18.

A number that is a factor of two or more numbers is a **common factor** of those numbers. The largest common factor of two or more numbers is called the **greatest common factor (GCF)**.

A fraction is in its **simplest form** if 1 is the only number that will divide both the numerator and the denominator. If a fraction is not in simplest form, you can reduce it. **Simplify** means to reduce a fraction to the lowest **equivalent**, or equal fraction. To simplify a fraction, divide both the numerator and the denominator by the greatest common factor. Then, rewrite the fraction.

Finding Greatest Common Factor (GCF)



What is the greatest common factor of 8 and 12? factors of 8: 1, 2, 4, and 8 factors of 12: 1, 2, 3, 4, 6, and 12 The greatest common factor of 8 and 12 is 4.

Simplifying Fractions



Divide both the numerator and the denominator by the greatest common factor.



Greatest common factor of 18 and 42 is 6.

Problems to Try

- a. What is the greatest common factor of 36 and 48? Answer: 12
- **b.** Simplify $\frac{3}{15}$. Answer: $\frac{1}{5}$

Real-World Connection

You use fractions every time you look at a clock. The expression "half past the hour" in-

cludes a fraction $\left(\frac{1}{2}\right)$.

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Lesson Introduction: Converting Mixed Numbers and Improper Fractions

Common Core State Standard

- Fifth Grade: 5.NF.3-7
- Sixth Grade: 6.NS.1, 6.NS.4
- Seventh Grade: 7.NS.1
- Eighth Grade: 8.NS.1

Objective

- Convert mixed numbers to improper fractions.
- Convert improper fractions to mixed numbers.

Vocabulary

convert, denominator, improper fraction, mixed numbers, numerator, whole number

Overview

A **mixed number** is a whole number and a fraction. A **whole number** is a counting number. An **improper fraction** is a fraction in which the numerator is greater than the denominator. The **numerator** of a fraction is the top number, and the **denominator** is the bottom number. It is easy to **convert** or change a mixed number to an improper fraction or to change improper fractions to mixed numbers.

Converting Mixed Numbers to Improper Fractions



Lesson Introduction: Multiplying a Fraction by a Fraction

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.4, 5.NF.6
- Sixth Grade: 6.NS.1
- Seventh Grade: 7.NS.2, 7.NS.3
- Eighth Grade: 8.NS.1

Vocabulary

denominators, fraction, numerators, simplify

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. To multiply fractions, first multiply the **numerators**, or the top numbers. Then multiply the **denominators**, or the bottom numbers. If possible, simplify the answer. **Simplify** means to rewrite the answer in its lowest terms.

Objective

lowest terms.

Multiply a fraction by a fraction.

Simplify the product of two fractions to

Multiplying a Fraction by a Fraction



Problems to Try

a. $\frac{1}{3} \times \frac{1}{2} =$ **b.** $\frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} =$

Answer: $\frac{1}{6}$

Answer: $\frac{4}{24} = \frac{1}{6}$

Real-World Connection

The gas gauge in a car is based on fractions. If the gas tank on a car holds 20 gallons of gas, and the gauge indicates $\frac{1}{4}$ tank, you have 5 gallons of gas.



Lesson Introduction: Multiplying Fractions and Whole Numbers

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.4, 5.NF.6
- Sixth Grade: 6.NS.1
- Seventh Grade: 7.NS.2, 7.NS.3
- Eighth Grade: 8.NS.1

Objective

- Multiply a fraction with a whole number.
- Simplify the product of two fractions to lowest terms.

Vocabulary

denominator, fraction, numerators, simplify, whole number

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form a/b. A **whole number** is a counting number, such as 1, 2, or 3.

To multiply fractions and whole numbers, first rewrite the whole number as a fraction by placing the whole number over the number 1. Next, multiply the **numerators**, or the top numbers. Then multiply the **denominators**, or the bottom numbers. If possible, simplify the answer. **Simplify** means to rewrite the answer in its lowest terms.

Multiplying Fractions and Whole Numbers



$$\frac{1}{4} \times 12 =$$
 b. $7 \times \frac{2}{5} =$

 Answer: 3
 Answer: $2\frac{4}{5}$

Real-World Connection

а.

Several fast-food restaurants sell quarter-pound hamburgers. If you understand fractions, you can figure out how many ounces of beef you will get in your hamburger.

Lesson Introduction: Multiplying Mixed Numbers

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.4, 5.NF.6
- Sixth Grade: 6.NS.1

Eighth Grade: 8.NS.1

• Seventh Grade: 7.NS.2, 7.NS.3

Objective

- Multiply mixed numbers.
- Convert mixed numbers to improper fractions.
- Convert improper fractions to mixed numbers.
- Simplify products to lowest terms.

Vocabulary

convert, denominators, fraction, improper fraction, mixed number, numerators, simplify, whole number

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. A **whole number** is a counting number, such as 1, 2, or 3. A **mixed number** is a whole number and a fraction. A mixed number can be converted to an improper fraction. An **improper fraction** is a fraction in which the numerator is greater than the denominator. To **convert** means to change.

An improper fraction can be converted to a mixed number by dividing the numerator by the denominator. To convert a mixed number to an improper fraction, first multiply the whole number by the **denominator** or bottom number of the fraction. Add the **numerator** or top number to the product. The sum is the new numerator in your improper fraction. Keep the denominator the same.

To multiply mixed numbers, first convert the mixed numbers to improper fractions. If one of the numbers being multiplied is a whole number, rewrite the whole number as a fraction by placing the whole number over the number 1. Next, multiply the numerators. Then multiply the denominators. If possible, simplify the answer. **Simplify** means to rewrite the answer in its lowest terms.

Multiplying Mixed Numbers



When you take your car to an automotive shop, the mechanics might use a $\frac{3}{16}$ -inch wrench or 1 $\frac{7}{8}$ -inch socket to repair your car.

Lesson Introduction: Multiplying Fractions and Mixed Numbers: A Shortcut

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.4, 5.NF.6
- Sixth Grade: 6.NS.1
- Seventh Grade: 7.NS.2, 7.NS.3
- Eighth Grade: 8.NS.1

Objective

• Cross-cancel to make multiplying fractions easier to work with.

Vocabulary

cross-cancel, fraction

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. When working with fractions, you can cross-cancel to make the problem easier to work with. Instead of simplifying the fraction at the end of the problem, you can cross-cancel before you multiply. **Cross-cancel** means to simplify before you multiply.

Cross-Cancel



a.
$$\frac{2}{3} \times \frac{7}{10} =$$
 Answer: $\frac{7}{15}$ **b.** $1\frac{5}{6} \times \frac{5}{11} \times 2\frac{1}{4} =$ **Answer:** $1\frac{7}{8}$

Real-World Connection

Recipes often call for $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$ -cup measurements of ingredients. If you decide to double or triple your favorite chocolate chip cookie recipe, you will need to have an understanding of how to multiply fractions.



Lesson Introduction: Reciprocals

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.7 Sixth Grade: 6.NS.1
- Seventh Grade: 7.NS.2, 7.NS.3
- Eighth Grade: 8.NS.1

Vocabulary

fraction, denominator, numerator, improper fraction, mixed number, reciprocal, whole number

and mixed numbers.

Objective

Find the reciprocal of whole numbers, fractions,

Overview

One number is the **reciprocal** of another if their product is 1. A **whole number** is a counting number, such as 1, 2, or 3. A fraction expresses a whole divided into any number of equal parts. It is a number usually expressed in the form a/b. An **improper fraction** is a fraction in which the numerator (the top number) is greater than or equal to the denominator (the bottom number). A **mixed number** is a whole number and a fraction.

Finding the Reciprocal

The reciprocal of a whole number is obtained by placing the whole number over the number 1, making a fraction. Next, invert the fraction or flip the numerator and the denominator.

The reciprocal of 4 is
$$\frac{1}{4}$$
, since $\frac{4}{1} \times \frac{1}{4} = 1$.

The reciprocal of a fraction is obtained by inverting the fraction or by flipping the numerator and the denominator.

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, since $\frac{3}{4} \times \frac{4}{3} = 1$.

The reciprocal of a mixed number is obtained by converting the mixed number to an improper fraction and then inverting the improper fraction.

The reciprocal of $1\frac{1}{2}$ is $\frac{2}{3}$, since $1\frac{1}{2} = \frac{3}{2}$ and $\frac{3}{2} \times \frac{2}{3} = 1$.

Problems to Try

Examples

- **a.** Find the reciprocal of 10. **Answer:** $\frac{1}{10}$ **b.** Find the reciprocal of $\frac{1}{5}$. **Answer:** $\frac{5}{1}$
- **c.** Find the reciprocal of $2\frac{4}{5}$. **Answer:** $\frac{5}{14}$

Real-World Connection

Musical notes tell a musician which notes to play, when to play them, and how to play them. Examples of musical notes are $\frac{1}{1}$ (whole), $\frac{1}{2}$ (half), $\frac{1}{4}$ (quarter), $\frac{1}{8}$ (eighth), $\frac{1}{16}$ (sixteenth), and the very rare $\frac{1}{32}$ (thirty-second).

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Lesson Introduction: Dividing by Fractions

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.7Sixth Grade: 6.NS.1
- Sixin Grade: 6.NS. I
- Seventh Grade: 7.NS.2, 7.NS.3

• Eighth Grade: 8.NS.1

Vocabulary

denominator, fraction, inverting, numerators, reciprocal, simplify, whole number

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. One number is the **reciprocal** of another if their product is 1. The reciprocal of a fraction is obtained by **inverting** the fraction or by flipping the **numerator** (top number) and the **denominator** (bottom number). **Simplify** means to rewrite the answer in its lowest terms.

When dividing a fraction by a fraction, first invert the second fraction. This fraction is now a reciprocal. Change the division sign to a multiplication sign. Multiply the two fractions. If possible, simplify the answer.

Dividing Fractions

Example



Problems to Try

	1	1	3		1	2	1
a.	4 ÷	3 =	Answer: $\overline{4}$	b.	<u>2</u> ÷	5 =	Answer: 1 4

Real-World Connection

Veneer is a wood product made of thin layers of wood glued together. It is commonly used in furniture manufacturing. Woodworkers measure the thickness of veneers in fractions such as

$$\frac{3}{32}$$
 or $\frac{1}{40}$ of an inch.

Objective

- Find the reciprocal of a fraction.
- Divide a fraction by a fraction.
- Simplify answer to lowest terms.

Divide a whole number by a mixed number.

• Divide a mixed number by a whole number.

Lesson Introduction: Dividing Whole Numbers and Mixed Numbers

Objective

Common Core State Standard

- Fifth Grade: 5.NF.3, 5.NF.7Sixth Grade: 6.NS.1
- Seventh Grade: 7.NS.2, 7.NS.3
- Eighth Grade: 8.NS.1

Vocabulary

convert, fraction, reciprocal, improper fraction, invert, mixed number, whole number

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. A **whole number** is a counting number, such as 1, 2, or 3. A **mixed number** is a whole number and a fraction. A mixed number can be converted to an improper fraction. An **improper fraction** is a fraction in which the numerator is greater than the denominator. To **convert** means to change.

Dividing fractions requires using the reciprocal. One number is the **reciprocal** of another if their product is 1. When you turn the fraction upside down or **invert** the fraction, you have a reciprocal of the original fraction.

Dividing Whole Numbers and Mixed Numbers



Place the whole number over the denominator of 1. Convert the mixed number to an improper fraction. Invert the improper fraction and change the sign to multiplication. Convert the answer back to a mixed number.

$$14 \div 2\frac{1}{2} = \frac{14}{1} \div \frac{5}{2} = \frac{14}{1} \times \frac{2}{5} = \frac{28}{5} = 5\frac{3}{5}$$

Dividing Mixed Numbers

Convert the mixed numbers to improper fractions. Invert the second improper fraction and change the sign to multiplication. Simplify before multiplying, if possible. Convert the answer back to a mixed number.

$$3\frac{3}{5} \div 2\frac{2}{3} = \frac{18}{3} \div \frac{8}{3} = \frac{18}{3} \times \frac{3}{8} = \frac{9}{4} = 2\frac{1}{4}$$

Problems to Try

Example

a.
$$4 \div 1\frac{1}{2} =$$
 Answer: $2\frac{2}{3}$

b.
$$5\frac{1}{4} \div 1\frac{3}{4} = A$$

Answer: 3

Real-World Connection

In U.S. currency, a penny is one-hundredth of a dollar, a nickel is onetwentieth of a dollar, a dime is one-tenth of a dollar, and a quarter is one-fourth of a dollar.



Lesson Introduction: Converting Decimals to Fractions

Common Core State Standard

- Fifth Grade: 5.NBT.3
- Eighth Grade: 8.NS.1

Vocabulary

convert, decimal, decimal point, fraction

Overview

A **decimal** is a fractional number written after a period called a **decimal point**. A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. It is easy to **convert** or change a decimal to a fraction.

Objective

to fractions.

Convert decimals through thousandths

Converting Decimals to Fractions



Problems to Try

Write the fraction for each decimal.



A familiar use of decimals is with money. Examples of decimals as money are \$0.47, \$6.50, and \$20.00.



Lesson Introduction: Converting Fractions to Decimals

Common Core State Standard

- Fifth Grade: 5.NBT.3
- Eighth Grade: 8.NS.1

Objective

• Convert fractions to decimals through thousandths.

Vocabulary

decimal, decimal point, fraction, convert

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. A **decimal** is a fractional number written after a period called a **decimal point**. It is easy to **convert** or change a fraction to a decimal.

Converting Fractions to Decimals



Problems to Try

Write the fraction for each decimal.



Real-World Connection

Map navigation systems give directions using decimals and fractions, such as "Go 7.9 (7 $\frac{9}{10}$) miles."



Lesson Introduction: Dividing to Convert Fractions to Decimals

Common Core State Standard

- Fifth Grade: 5.NBT.3, 5.NBT.4, 5.NBT.5, 5.NBT.7
- Sixth Grade: 6.NS.3
- Seventh Grade: 7.NS.2
- Eighth Grade: 8.NS.1

Objective

• Use division to express a fraction as a decimal.

Vocabulary

decimal, decimal point, denominator, fraction, numerator, quotient

Overview

A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. A **decimal** is a fractional number written after a period called a **decimal point**. Division can be used to express a fraction as a decimal. To express a fraction as a decimal, divide the **numerator** (top number) by the **denominator** (bottom number). The answer to a division problem is the **quotient**. The quotient is the decimal version of the fraction.

Dividing to Convert Fractions to Decimals

Examples	To express $\frac{1}{5}$ as a decimal, divide 1 by 5. Divide until the remainder is zero.	To express $\frac{2}{3}$ as a decimal, divide 2 by 3. Divide until you can round the quotient to the nearest thousandth.
	$ 5) 1.0 \\ - 10 \\ 0 $	$\frac{0.6666}{3} = 0.667$ $\frac{-18}{20}$
Problems to Try		$-\frac{18}{20}$
a. Divide to ex dividing unt	press $\frac{3}{16}$ as a decimal. Continue il the remainder is zero.	$ \begin{array}{r} \underline{10} \\ \underline{20} \\ \underline{-18} \\ \underline{2} \end{array} $
Answer: 0	.1875	

b. Divide to express $\frac{2}{9}$ as a decimal. Divide until you can round the quotient to the nearest thousandth. **Answer:** 0.222

Real-World Connection

When you buy or sell anything in the United States, you are using decimals and fractions. A dime is 1/10 of a dollar or \$0.10.

Lesson Introduction: Converting Fractions to Percents

Common Core State Standard

- Fifth Grade: 5.NF.3
- Sixth Grade: 6.RP.3
- Seventh Grade: 7.RP.3
- Eighth Grade: 8.NS.1

Vocabulary

convert, fraction, percent

Overview

Fractions and percents are basically the same thing—both represent "parts of a whole" or "a part of." A **fraction** expresses a whole divided into any number of equal parts. It is a number usually expressed in the form *a/b*. **Percent** means hundredths or out of 100. The symbol for percent is %. It is easy to convert fractions to percents. **Convert** means to change.

Objective

Convert a fraction to a percent.

Converting Fractions to Percents



Problems to Try

a.	Write a percent for $\frac{75}{100}$.	Answer: 75%	b.	Write a percent for $\frac{1}{5}$.	Answer: 20%
-					

c. 53 of the 100 students are girls. What percent are girls? **Answer:** 53%

Real-World Connection

Most states have a sales tax. This is a tax paid by the consumer when they buy something. Sales tax on a new car can amount to more than most people expect. An 8% sales tax on a soda isn't too much, but 8% on a \$40,000 car will cost you \$3,200!

Lesson Introduction: Probability

Vocabulary

probability

Overview

Probability is the chance or likelihood that an event will happen. The probability of an event happening can be written as a fraction.

Probability of an event occurring = Number of ways it can happen Total number of possible outcomes

Probability



Problems to Try

- a. If you toss a coin 100 times, how many times will heads come up? Answer: 50
- b. What is the probability of drawing a green marble from the bag of marbles above?

Answer: $\frac{1}{7}$

Real-World Connection

The population of the United States is about 310 million. In the United States, an average of 40 people are killed by lightning each year. The chance of being killed by lightning in the United States is equal to 40/310 million.

Lesson Introduction: Meaning of Ratios

Common Core State Standard

- Fifth Grade: 5.NBT.3
- Sixth Grade: 6.RP.1-3
- Seventh Grade: 7.SP.1-3
- Eighth Grade: 8.NS.1

Vocabulary

ratio, rate, expressed

Overview

A **ratio** is a pair of numbers that expresses a rate or comparison. It is a comparison of the size of one number to the size of another number. A **rate** is a ratio that expresses how long it takes to do something, such as traveling a certain distance. Ratios can be **expressed** or written in different ways.

Objective

Describe ratios with numbers.

Writing Ratios



Write a ratio expressing the number of squares to circles as a fraction (label numbers), decimal, and percent.



Answer: $\frac{\text{squares}}{\text{circles}} \xrightarrow{} \frac{1}{3}$, 0.33, 33%

Real-World Connection

Pharmacists must be familiar with ratios and rates. Prescription dosages are based on a ratio of medicine to body mass and to frequency of ingestion.

Lesson Introduction: Finding Equivalent Ratios

Common Core State Standard

- Fifth Grade: 5.NBT.3
- Sixth Grade: 6.RP.1-3
- Seventh Grade: 7.SP.1-3
- Eighth Grade: 8.NS.1

• Write a ratio and label the numbers.

• Use multiplication to find equivalent ratios.

Vocabulary

ratio, rate, equivalent

Overview

A **ratio** is a pair of numbers that expresses a rate or comparison. It is a comparison of the size of one number to the size of another number. A **rate** is a ratio that expresses how long it takes to do something, such as traveling a certain distance. **Equivalent** ratios are ratios that are equal.

Finding Equivalent Ratios



Problems to Try

The team scored 4 goals in 7 games. List equal ratios to find how many goals the team will score

	_	goals —	<u>4</u>	8	<u>12</u>		<u>16</u>	<u>20</u>	<u>24</u>	
in 42 soccer games.	Answer:	games	7 =	14	= 21	=	28 =	<u>35</u> :	= 42	

Real-World Connection

Ratios are important in education. Research suggests that low student-to-teacher ratios promote higher achievement levels. A 15:1 student-to-teacher ratio is better than a 25:1 ratio.

Lesson Introduction: Finding Missing Numbers and Cross-Products of Ratios

Common Core State Standard

- Fifth Grade: 5.NBT.3
- Sixth Grade: 6.RP.1-3
- Seventh Grade: 7.SP.1-3
- Eighth Grade: 8.NS.1

Objective

- Find the missing number in a ratio.
- Cross multiply to find if two ratios are equal.

Vocabulary

equivalent, rate, ratio

Overview

A **ratio** is a pair of numbers that expresses a rate or comparison. It is a comparison of the size of one number to the size of another number. A **rate** is a ratio that expresses how long it takes to do something, such as traveling a certain distance. **Equivalent** ratios are ratios that are equal.

Finding Missing Numbers

Example	Find <i>n</i> in $rac{1}{4}$	$r = \frac{n}{8}$	How many times does 4 Multiply that number tim numerator 1.	divid	e into 8? e	1×2 $\frac{1}{4} = \frac{2}{8}$
Cross-Produ	ct of Ratios			_		4 x 2
Example	Cross multiply $\frac{2}{3}$ $\xrightarrow{6}$ $\frac{6}{9}$	y to find i = <u>18</u> 18	if two ratios are equal.			
	The two ratio	s are equ	vivalent because their cros	ss-pro	oducts are t	the same.
Problems to 7 a. Find <i>n</i> fo $\frac{3}{9} = \frac{6}{n}$	Try r the ratio. b. Answer: 18	Marilyr 4 slice	n uses 3 eggs to make s of French toast. How eggs will she use for 12	c.	Are the ra equal? Fin products	$\frac{3}{4}$ and $\frac{6}{8}$ nd the cross- to help you

Real-World Connection

Jobs in the medical field involve using equipment to measure ratios and rates such as blood pressure and pulse.

Answer: 9



decide.

Answer: ves

slices of French toast?